

THREE-NUCLEON PORTRAIT WITH PION

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We report on recent results obtained by the above collaboration on the collision processes involving three nucleons, where we pay particular attention on the dynamical role of the pion. After discussing the case at intermediate energies, where real pions can be produced and detected, we have considered the case at lower energies, where the pions being exchanged are virtual. The study has revealed the presence of some new pion-exchange mechanisms, which leads to a new three-nucleon force of tensor structure. Recently, the effect of this tensor three-nucleon force to the spin observables for neutron-deuteron scattering at low energy has been analyzed, and will be briefly reviewed.

1 Introduction

A crucial aspect in modern three-nucleon dynamics concerns the inclusion in the system of additional mesonic aspects which cannot be conveniently described by a conventional (*e.g.*, meson-exchange) two-nucleon potential. The standard approach to this problem introduces three-nucleon forces ($3NF$), which effectively describe the addition of such irreducible mesonic contributions. Such “ $3NF$ ” approaches turned out to be successful in curing the problem of the underbinding of the three-nucleon bound state, but failed to explain the puzzle of the vector analyzing powers (“the A_y puzzle”) in nucleon-deuteron scattering at low energy.

Recently, with the aim to explain the A_y puzzle, there have been various approaches which introduced $3NF$ of new structure, in the attempt to characterize, more or less effectively, some refined aspects of the meson dynamics which were not yet contemplated in the traditional $3NF$ expressions. The starting point of these descriptions is characterized by a Hamiltonian which is restricted in the three-nucleon space, wherein the mesonic aspects have

been integrated out from the very beginning via the introduction of $3NF$ -like contributions.

In contrast, we have considered an approach which starts from the employment of the full four-body dynamics of the one-pion three-nucleon system. The method is based on a generalization to the one-pion three-nucleon problem of the rigorous four-body theory of Grassberger-Sandhas-Yakubovsky¹, this generalization being developed by one of the authors². Such generalization turned out to be a highly nontrivial task, and a physically sound approximation scheme has been recently derived for the resulting dynamical equations³.

To the lowest order, such approximation scheme reproduces the diagrams leading to the $3NF$ *à la* Tucson-Melbourne⁴. However, at the same level of approximation, other irreducible diagrams of different structure appeared. In particular, a recent detailed analysis⁵ for one class of such diagrams showed the appearance of a new three-nucleon force of tensor structure. This $3NF$ component is generated by the underlying one-pion exchange diagram where one of the two nucleons interacts with the third one while the pion is being exchanged. In a conventional $3NF$ approach, where the meson degrees of freedom are integrated out from the very beginning, such diagram is usually neglected because of the presence of a cancellation effect involving meson-retardation effects of the combined exchange of two pions. However, by treating explicitly the pion dynamics, it was found⁵ that this cancellation is incomplete, and a 30% effect survives once these mesonic retardations are taken into account. In a recent work⁶, the effect of this new $3NF$ of tensor structure has been studied in neutron-deuteron scattering at low energy.

2 Pion production from few-nucleon collisions

Here we summarize the results obtained on pion production from collisions involving few-nucleon systems. For full details, we refer to the original papers^{7–11}. The starting point is the low-energy Lagrangian coupling the pion (Φ) and nucleon (Ψ) fields,

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \frac{f_{\pi NN}}{m_\pi} \bar{\Psi} \gamma^\mu \gamma^5 \vec{\tau} \Psi \cdot \partial_\mu \vec{\Phi} \\ & - 4\pi \frac{\lambda_I}{m_\pi^2} \bar{\Psi} \gamma^\mu \vec{\tau} \Psi \cdot \left[\vec{\Phi} \times \partial_\mu \vec{\Phi} \right] - 4\pi \frac{\lambda_O}{m_\pi} \bar{\Psi} \Psi \left[\vec{\Phi} \cdot \vec{\Phi} \right] . \end{aligned} \quad (1)$$

The choice of the two constants λ_I and λ_O is consistent with the pion-nucleon scattering lengths; however the kinematics for the pion-production mechanism

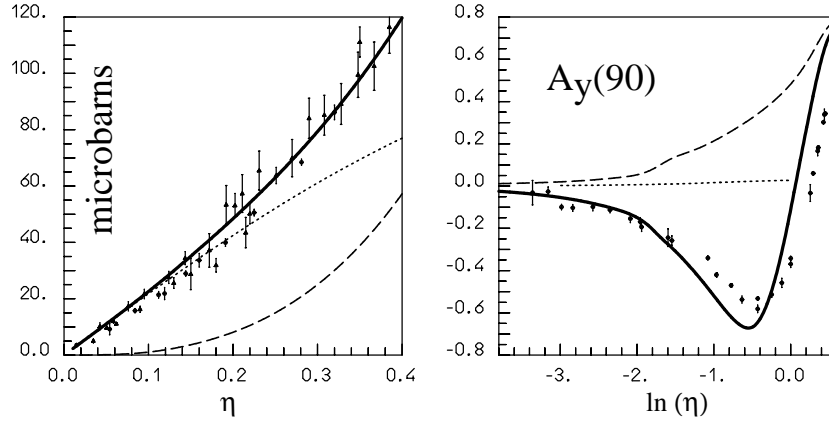


Figure 1. Total cross section (*left*) and proton analyzing power (*right*) at 90° for the $pp \rightarrow \pi d$ reaction. η is the dimensionless pion c.m. momentum in units of pion masses. The solid line corresponds to the complete model. The dotted line corresponds to the mechanisms implied by the low-energy Lagrangian of Eq. (1). The dashed line represents the p -wave (πN) mechanisms and includes the Δ effects. The data shown are Coulomb corrected experimental data, which are referenced elsewhere⁸.

requires an off-shell extrapolation of the two constants, which can be conveniently obtained by representing the corresponding four-leg vertices in terms of a combination of exchanges of heavier mesons, ρ , σ and some additional effective short-range effects¹².

The description of the pion production process in terms of this Lagrangian is however not sufficient, because of the strong coupling of the πN system to the Δ channel. Therefore, one must consider, in addition, the mechanisms triggered by the $\pi N \Delta$ coupling and by the $V(\Delta N - NN)$ transition potential. As shown in Fig. 1, this occurs even around threshold, since the structure of A_y is governed by the interference effects between the Δ and non- Δ mechanisms. The results obtained^{10,11} for pion production from pd collision show that this interplay is even more important here, and the relevance of the various production mechanisms shows up already at the level of the integral cross section (see the contribution to these Proceedings by M. Viviani,¹³ Fig. 2). In particular, the off-shell effects in the isoscalar channel turned out to be very important, for the normalization of the production cross-section, as well as for describing the spin observables at threshold.

3 Pion dynamics and new three-nucleon-force diagrams

The explicit pion dynamics can be included in the $3N$ system² by generalizing the rigorous $3N$ approach in terms of AGS equations¹⁴. One advantage is consistency between the treatment of such mesonic aspects, and the method of solution of the three-nucleon problem with a given $2N$ potential. Starting from this result a practical approximation scheme has been derived³, which treats perturbatively the four-body dynamics of the pion-three-nucleon system. To the lowest order, the approach leads to $3NF$ diagrams, which can be incorporated into an effective two-cluster $3N$ equation:

$$X_{ss'}^{(2)} = Z_{ss'}^{(2)} + \sum_{s''} Z_{ss''}^{(2)} T_{s''}^{(2)} X_{s''s'}^{(2)} \quad (2)$$

where the driving term is given by two contributions

$$Z_{ss'}^{(2)} = Z_{ss'}^{AGS} + Z_{ss'}^{3NF}. \quad (3)$$

The first term represents the standard nucleon-exchange diagram between different correlated pairs of nucleons, while the second contains irreducible exchange diagrams involving the pion (see Fig. 2). Beside the standard $3NF$ diagram –the first diagram in the figure– where the pion rescatters with the

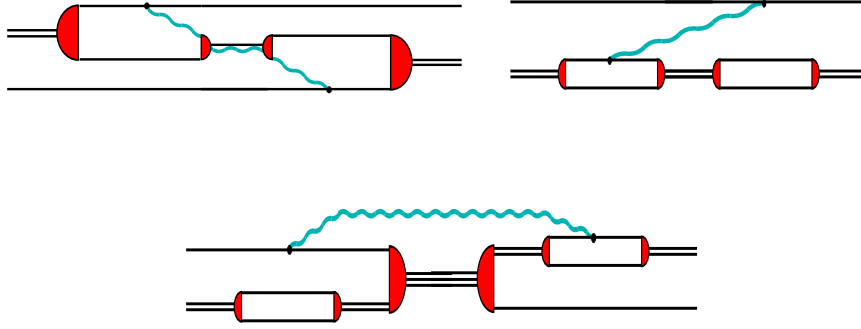


Figure 2. Irreducible $3NF$ contributions generated –at the lowest order– by the pion dynamics in the AGS equation. The straight lines are nucleons, the wavy line represents the pion.

nucleon while being “in flight” (it represents the basic diagram for the construction of the “standard” three-nucleon force), we obtain at the same order two additional diagrams which lead to $3NF$ of different structure. At the present stage we have considered the consequences implied by the second diagram of Fig. 2, representing a $2N$ rescattering process while the pion is “in flight”. The combined effect of the two $3NF$ terms implied by the first two diagrams in Fig. 2 have been studied also in a schematic one-dimensional model¹⁵. Further work is needed in order to consider the last type of diagrams shown in figure.

4 Tensor three-nucleon force

The tensor structure of the $3NF$ implied by the second diagram shown in Fig.2 has been discussed recently⁵. The explicit expression is

$$\begin{aligned}
 V_3^{3N}(\mathbf{p}, \mathbf{q}, \mathbf{p}', \mathbf{q}'; E) = & \frac{f_{\pi NN}^2(Q)}{m_\pi^2} \frac{1}{(2\pi)^3} \\
 & \times \left[\frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{Q})(\boldsymbol{\sigma}_3 \cdot \mathbf{Q})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3) + (\boldsymbol{\sigma}_2 \cdot \mathbf{Q})(\boldsymbol{\sigma}_3 \cdot \mathbf{Q})(\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)}{\omega_\pi^2} \right] \\
 & \times \frac{\tilde{t}_{12}(\mathbf{p}, \mathbf{p}'; E - \frac{q^2}{2\nu} - m_\pi)}{2m_\pi} \\
 & + \frac{f_{\pi NN}^2(Q)}{m_\pi^2} \frac{1}{(2\pi)^3} \frac{\tilde{t}_{12}(\mathbf{p}, \mathbf{p}'; E - \frac{q'^2}{2\nu} - m_\pi)}{2m_\pi} \\
 & \times \left[\frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{Q})(\boldsymbol{\sigma}_3 \cdot \mathbf{Q})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3) + (\boldsymbol{\sigma}_2 \cdot \mathbf{Q})(\boldsymbol{\sigma}_3 \cdot \mathbf{Q})(\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)}{\omega_\pi^2} \right].
 \end{aligned} \tag{4}$$

The momenta \mathbf{p}, \mathbf{q} represent the Jacobi coordinates of the pair “12”, and spectator “3” in the incoming state, and similarly \mathbf{p}', \mathbf{q}' are for the outgoing channel. E is the $3N$ energy and \mathbf{Q} is the momentum carried by the pion, $\mathbf{Q} = \mathbf{q} - \mathbf{q}'$. The tensor structure arises because of the presence of the One-Pion-Exchange term between the spectator nucleon and the pair, while the pair correlation is described by means of the $2N$ t -matrix, evaluated at the off-shell energy, as required from the kinematics for the subsystems.

The above expression has been obtained by taking into account the dynamical effects of one single pion. It is clear, however, that one should expect that the combined exchanges of more than one pion will have some additional effects. However, the irreducible diagrams generated by the combined exchange of two pions –at leading order– cancel out against the mesonic retardation effects of the reducible contributions –at second order– of the Born dia-

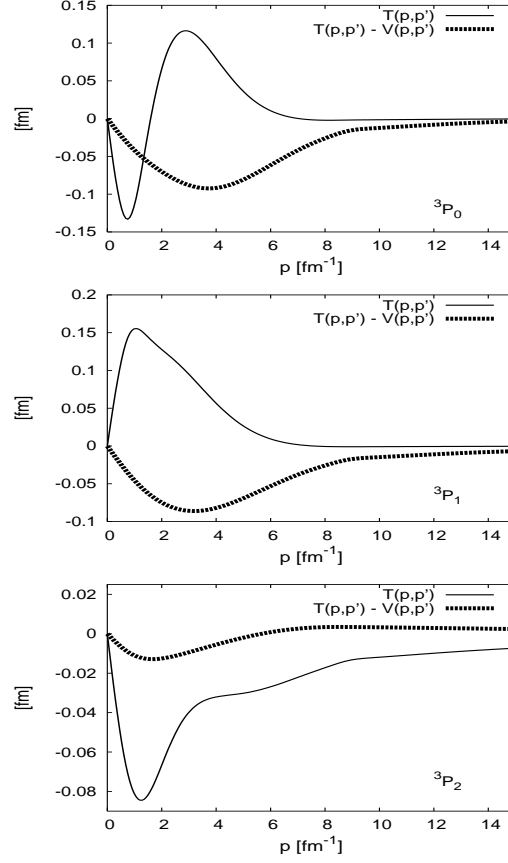


Figure 3. Comparison between the subtracted (thick-dashed) and unsubtracted (thin-solid) t -matrix in the triplet p -waves for the CD-Bonn potential¹⁶. The t -matrix has been calculated at $E = -150$ MeV, and for a fixed momentum p' of 0.089 fm^{-1} .

gram. This cancellation has been observed by various authors^a, but it has been also argued recently⁵ that this cancellation is incomplete, and leads in Eq. (4) to a *subtracted* t -matrix: $\tilde{t}_{12}(\mathbf{p}, \mathbf{p}'; E - \dots) = t_{12}(\mathbf{p}, \mathbf{p}'; E - \dots) - v_{12}(\mathbf{p}, \mathbf{p}')$. In Fig. 3 it is shown that this subtracted \tilde{t} -matrix is not negligible in the relevant kinematical region, if compared to the unsubtracted t -matrix. The

^aSee, *e.g.*, the references contained here⁵

figure refers to the Bonn CD potential, however we obtained similar results for other modern potentials, and also in other $2N$ states.

Fig. 4 is taken from a recent preprint⁶ (which we refer to for all details), and shows the results obtained when the $3NF$ herein discussed is included, with respect to a calculation with the Paris $2NF$, only. As $2N$ interaction we have considered the high-rank parameterization of this interaction known as *PEST*, developed by the Graz group¹⁸.

5 Summary

Pion production reactions have been studied by us theoretically with a combination of mechanisms involving meson exchanges and Δ excitations. The results obtained showed that such combination is appropriate for describing unpolarized and polarized observables at low energies.

Also, an approach for the treatment of pion dynamics has been developed, by taking into account the complete four-body dynamics of the pion and three nucleon system. The resulting equations have been treated with an approximation scheme which leads, to the lowest order, to effective three-nucleon interactions. It has been shown that one class of diagrams leads to a tensor-like three-nucleon potential. It has been argued how this new term

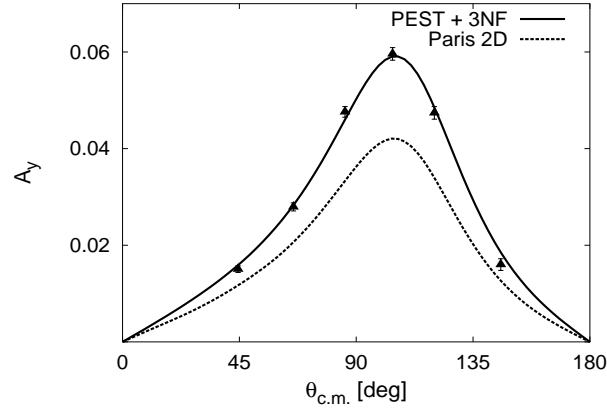


Figure 4. A_y for nd scattering at 3 MeV (Lab). Solid line with the tensor $3NF$ herein discussed. Dashed line without this contribution. Data from Ref.¹⁷.

could explain the puzzle of the nucleon-deuteron vector analyzing powers.

Acknowledgments

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